Measure theoretical approach for chaotic dynamical systems

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1 ABSTRACT

We study chaotic dynamical systems from measure theoretical view point. Let f be a observable defined on a phase space X. We are interested in time evolutions of it $f, fT, fT^2, \ldots, fT^n \ldots$, i.e., their asymptotic distribution behaviors and limiting distributions of their partial sums. In ergodic theory, there are several characterizations of dynamical systems with high degrees of randomness. Typical examples are hyperbolic systems which can be reduced to Markov process through a nice coarse-graining. On the other hand, in probability, weak independent process are studied by many people. However most of those are established only for Markov processes and strongly independent processes.

In this talk, we introduce a technique for connecting random process come from dynamical systems and random process in probability theory. It allows us to characterize sensitive dependence of dynamical systems on initial distributions. Further we show several chaotic dynamical systems in the above sense which are not necessarily hyperbolic. One of such Mathematical models, a class of mappings providing nice measure theoretical structure is the following : Let X be a bounded domain of $\mathbf{R}^{\mathbf{d}}$ and T a transformation of X. Assume that there exists a generating countable partition $Q = \{X_a\}_{a \in I}$ of X s.t. $T|_{X_a} : X_a \to TX_a$ is a C^1-diffeomorphism. We define a cylinder set of rank n by

$$X_{a_1} \cap T^{-1} X_{a_2} \cap \ldots \cap T^{-(n-1)} X_{a_{n-1}}$$

if its interior is not empty and we denote by $X_{a_1 \ldots a_n}$. Let $\mathcal{U} = \{T^n X_{a_1 \ldots a_n} : \forall X_{a_1 \ldots a_n}, \forall n > 0\}$.

If $\mathcal{U}$ is a finite set, we call the quadruple $(T, X, Q = \{X_a\}, \mathcal{U})$ a piecewise invertible system with finite range structure (FRS).

Piecewise exapanding Markov maps are particular cases of the above system. Typical examples are number theoretical transformations.